Working paper

An approach to setting inflation and discount rates

Hugh Miller & Tim Yip
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1 Introduction

Setting inflation and discount assumptions is a core part of many actuarial tasks. AASB 1023 requires general insurance liabilities involve a central estimate (including an allowance for inflation) and then discounted at the risk free rate. However, there are a number of issues associated with setting these assumptions, including:

- What model should be adopted to fit the yield curve from observable risk free securities?
- How should a discount rate curve be extrapolated beyond the last observable risk free asset?
- What are appropriate long term discount and inflation rates?
- How should inflation rate assumptions vary with respect to changes in risk free rates?

This paper presents an approach to assumption setting that concretely addresses these questions, and provides a means of setting assumptions. The approach has been chosen to be faithful to the observed behaviour of the market and previous research on the topic. It describes the 'default' way that assumptions are set at Taylor Fry, while recognising that there is not a single approach that will be appropriate for every context.

2 Background

The approach presented in this paper relies heavily on previous research undertaken at Taylor Fry, as well as some other sources. The most important papers relied upon are described here. These papers in turn have more comprehensive lists of references for the interested reader.

2.1 Miller (2010)

The paper 'Towards a better inflation forecast' investigated inflation assumptions and the relationship between inflation and risk free forward rates. The most important conclusions were:

- Available industry forecasts such as Access Economics had some use in predicting inflation in the short term, but limited effectiveness in medium to long term prediction
- With respect to risk free rates, there is an index of models that range from the 'fixed rate' assumptions (long term inflation rate never changes) and the 'fixed gap' assumption (where a 1% increase in forward rates causes a 1% increase in inflation). This is indexed by the 'inflation parameter' θ, with $\theta = 0$ corresponding to fixed rate and $\theta = 1$ corresponding to fixed gap.
- A range of tests showed that the inflation parameters is closer to 0 than 1. Estimates for the parameter using a range of approaches gave a range of 0-0.3 for the inflation parameter for Average Weekly Earnings (AWE) inflation

- There is reasonable historical evidence that AWE and Labour Price Index (LPI) inflation is different across states. Higher rates for mining states (WA and QLD) appear justified, as are lower rates for some other states (NSW, VIC and TAS).

2.2 Mulquiney and Miller (2014)

The paper ‘A topic of interest – how to extrapolate the yield curve in Australia’ contained a detailed look at yield curve extrapolation, drawing from data in Australia and overseas. Relevant findings include:

- That the medium-long term forward rate (10 years say) has only partial ability to predict very long term rates (30 years and beyond). This can be thought of as long term reversion of the forward rate.
- The long term forward rate can be thought of as the combination of inflation expectations, real interest rate expectations, a risk premium and convexity adjustment. The combined estimate of 5.8% was presented as reasonable, and consistent with historical data.
- A linear reversion shape to the long term forward rate was judged reasonable, although other shapes are possible.
- The rate of reversion was observed to be slow, based on several different tests. Reversion by somewhere between 40 and 80 years was judged reasonable.

2.3 Intergenerational reports

The Australian Treasury regularly publishes the Intergenerational Report, which performs long range projections of the Australian economy. The most recent was published in 2015, and included the following assumptions:

- Long term bond rates of 6.0%
- Long term CPI inflation of 2.5%
- Long term AWE inflation of 4.0%

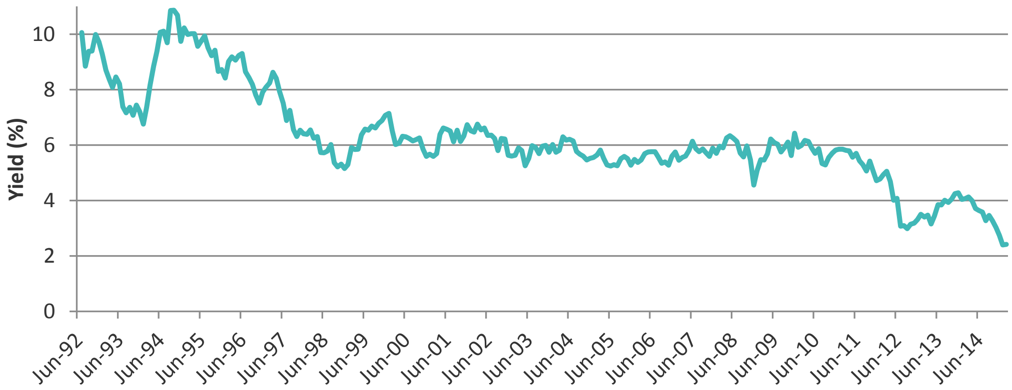
These assumptions are consistent with previous reports.

2.4 Other background

There have also been a number of recent events that influence assumption setting. First the last few years have seen very low bond rates – see Figure 1. This has increased the importance of a reversion assumption, as the 10 year bond rate is no longer close to the long term bond rate.

Secondly, the number and term of Australian government bonds on issue have increased. Whereas in the June 2005 there were 11 bonds on issue with maximum term 12 years, in March 2014 there were 21 bonds on issue with longest term 22 years. This increases the possible complexity of the yield curve shape and decreases the scope for a fast reversion of yields.

Figure 1 **10 year forward rate, Australian government bonds**[1]



3 Setting discount rates

3.1 Objectives of yield curve fitting

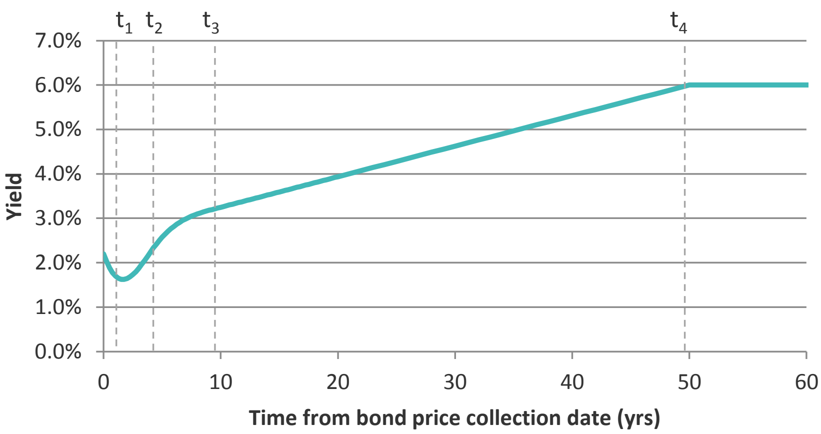
The main objective is to obtain a set of forward rates that is:

- Smooth: Generally viewed as a desirable feature. Additionally, non-smooth yield curves tend to present more arbitrage opportunity, so should be less frequent in practice
- Fits observable bond prices well: Each bond is viewed as the sum of zero coupon bonds. A good fit means that the price of those cash flows based on the forward rates is close to the observed bond price
- Reversion over the long term: The fitting model should be able to impose reversion to the long term rate once the term is beyond observable bond prices.

3.2 Adopted approach – constrained cubic spline model

The figure below illustrates our adopted shape. It comprises of a cubic spline shape between term 0 and term t_3, with to additional interior knots t_1 and t_2. Further it has a linear reversion between t_3 and t_4, with a constant forward rate beyond t_4.

Figure 2 **Schematic for adopted forward rate fitting shape**



[1] Source: RBA website, statistics series F16

In terms of equations, this is expressed as

$$f(x) = a + bx + d\lfloor x - 0\rfloor^3 + e\lfloor x - t_1\rfloor^3 + f\lfloor x - t_2\rfloor^3 + g\lfloor x - t_3\rfloor^3 \tag{1}$$

Here $\lfloor x\rfloor = x$ when $x > 0$ and $\lfloor x\rfloor = 0$ otherwise. Additionally, we impose the following constraints on the curve:

1. Reversion to the long term rate f^* at term t_4:

$$f(t_4) = a + bt_4 + d(x - t_4)^3 + e\lfloor t_4 - t_1\rfloor^3 + f(t_4 - t_2)^3 + g(t_4 - t_3)^3 = f^* \tag{2a}$$

   For this particular constraint we have set $f^* = 6.0\%$ and $t_4 = 50$, so that the forward rate

2. Linear reversion between terms t_3 and t_4. So in this region$f''(x) = 0$. Spitting constant and x components gives

$$d + e + f + g = 0 \quad \text{and} \tag{2b}$$

$$et_1 + ft_2 + gt_3 = 0 \tag{2c}$$

The equations (2a)-(2c) can be solved simultaneously to eliminate e, f and g from the minimisation:

$$e = \frac{\{f^* + a + b * t_4 + d * (t_4)^3\} + d\{-(t_3) * (t_4 - t_2)^3/(t_3 - t_2) + (t_2)(t_4 - t_3)^3/(t_3 - t_2)\}}{-(t_4 - t_1)^3 + (t_3 - t_1)(t_4 - t_2)^3/(t_3 - t_2) - (t_2 - t_1)(t_4 - t_3)^3/(t_3 - t_2)}$$

$$f = -\frac{e(t_3 - t_1) + d * (t_3 - 0)}{t_3 - t_2}$$

$$g = -(d + e + f)$$

Therefore the problem reduces to solving (1) for parameters a, b and d and knots t_1, t_2 and t_3

If $B_j$is the observed price of the jth bond, and $\hat{B}_j$ is the corresponding price estimate using the forward rate curve, then the parameters in equation (1) are chosen to minimise the weighted squared error:

$$\text{Error} = \textstyle\sum_j w_j\left(B_j - \hat{B}_j\right)^2 \tag{3}$$

Where the weight of each bond w_j is equal to $1/D_j^2$, with D_j the modified duration of bond j.

The unknown parameter and knots are chosen to minimise (3) using a non-linear optimiser. We have implemented this using the Solver functionality in Microsoft Excel.

3.3 Further comment on subjective assumptions

The two important assumptions in this fitting model are the choices for t_4 , the point at which the ultimate long term rate is achieved (here term 50 years), and the long term rate itself f^*, set to 6.0%. These have been selected with reference to the evidence in Section 2.

3.4 Alternative approaches for yield curve fitting

Before adopting the above cubic spline based fitting approach, we did consider a number of alternatives that are in the literature. While all could probably be amended to meet the objectives in 3.1, none did so 'out-of-the-box'. Further, differences in fitting approaches tend to be immaterial, apart from the assumptions related to extrapolation; as long as the curve is sufficiently flexible, it should give a reasonable fit of the observable securities. Other comparisons of approaches exist – see for instance Bolder and Gusba (2002).

Nelsen-Siegel functions

The Nelson-Siegel approach is to model the forward rate as a sum of exponential terms, which includes an exponential decay to a long term rate:

$$f(t) = \beta_0 + \beta_1 \frac{[1 - \exp(-t/\tau)]}{t/\tau} + \beta_2 \left(\frac{[1 - \exp(-t/\tau)]}{t/\tau} - \exp(-t/\tau) \right)$$

There are variations that include extra terms similar to the β_2 term (for example Svensson, 1994). In our attempts, when we used such an approach and imposed constraints on the long term rate and the length of time till reversion, there was not enough flexibility in the curve to ensure a good fit of forward rates on the short to medium parts of the curve.

Merrill Lynch Exponential spline model

This model is a relatively popular way of modelling discount factors (rather than forward rates) directly. The discount rate function (the price of a zero coupon bond with varying maturity) is expressed as a sum of hyperbolic functions.

$$d(t) = \sum_{k=1}^{K} \lambda_k \frac{1}{1 + k\alpha t}$$

This tends to produce good fits on the short to medium part of the curve, but poor extrapolation; indeed, without modification, forward rates always decay to zero in the long term.

Smith-Wilson Approach

The approach of Smith and Wilson (2001) is another popular approach that is now formally recognised in Solvency II in Europe. It includes decay to a long term rate, where the user must specify the level and rate of decay. For the observable part of the curve the discount rate function is expressed as a sum of exponential decay terms, with the number of terms equal to the number of securities. This approach tends to generate 'exact' fits, with the observed and predicted bond prices virtually identical. However, one consequence of this approach is a non-smooth forward rate curve, which appears to us undesirable.

4 Setting inflation rates

4.1 Our approach

Our approach to inflation forecasts is:

1. Adopting a third party econometric forecast in the short term (the first two years)
2. For the fifth year and beyond, adopting an inflation rate based on the estimated forward rate:

$$i(t) = i^* + \theta\{f(t) - f^*\}$$

3. For the third and fourth years, linearly blend between the two approaches.

The formula in the second point implies that, except for the initial few years), the shape of the inflation forecast will match the shape of the yield curve. This means that the inflation forecast will slowly revert to its ultimate long term rate i^* in a similar fashion to the yield curve. This approach has been adopted to be consistent with the findings presented in Section 2. The blending in the third and fourth years help avoid a cliff in forecasts, should the econometric and formula based forecasts materially differ.

In terms of explicit assumptions:

- We have selected i^* =2.5% for CPI inflation (the centre of the RBA target band), i^* =3.6% for LPI inflation (consistent with long run historical averages) and i^* =4.0% for AWE inflation (consistent with the intergenerational report and long run averages)
- We have selected θ =0.5 as the inflation parameter. Although higher than estimates in Miller (2010), it captures some of the sensitivity of inflation to nominal interest rates, and provides a balance between the 'fixed inflation' and 'fixed gap' extremes.
- We apply capping to the CPI forecast so that it does not exit the RBA target band (2.0% and 3.0%). That is, for CPI the adopted formula is slightly modified:

$$i(t) = \min(3.0\%, max(2.0\%, i^* + \theta\{f(t) - f^*\})).$$

- $f(t)$ and f^* are consistent with the previous section, with $f^* = 6.0\%$.

4.2 Modifiers for difference states

In addition to the Australia level forecasts in the previous subsection, we add modifiers to certain states:

- + 0.5% for LPI and AWE inflation for WA and QLD
- - 0.25% for LPI and AWE inflation for NSW, VIC and QLD.

Although these differentials were estimated in 2010, they have proven reasonably accurate over the past few years – see the table below. However, these factors will have to be reviewed regularly; changes to the nature of the mining boom will tend to influence appropriate choices for state based differences, and there is already some early evidence of WA inflation falling back to national levels (see for example Nicholls and Rosewell, 2015)

Table 1 **Historical AWE growth differentials for each state**[2]

Time period	State difference from national average								Aust
	NSW	VIC	QLD	SA	WA	TAS	NT	ACT	AWE
Jun 02 - Jun 06	-0.3%	-0.4%	0.4%	0.3%	1.3%	-1.3%	0.6%	1.2%	4.5%
Jun 06 - Jun 10	-0.7%	-0.5%	0.9%	-0.9%	1.8%	0.6%	-0.2%	0.4%	4.9%
Jun 10 - Jun 14	-0.6%	-0.4%	0.3%	-0.5%	1.5%	0.8%	1.5%	0.6%	3.9%

5 Conclusion

This paper presents a combined approach to inflation and discount rate assumption setting that should be appropriate for a wide range of actuarial contexts. Interested readers are encouraged to seek out the referenced papers, as well as contact Taylor Fry directly for further information.

6 References

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[2] ABS average weekly full time earnings, trend series